1.1 Making Conjectures: Inductive Reasoning

GOAL
Use reasoning to make predictions.

EXPLORE...

• If the first three colours in a sequence are red, orange, and yellow, what colours might be found in the rest of the sequence? Explain.

SAMPLE ANSWER

Here are three possible answers:

• If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, and purple. These colours are the primary and secondary colours seen on a colour wheel.

• If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, indigo, and violet. These colours are those of a rainbow.

• If the colour sequence is red, orange, and yellow, the rest of the sequence may repeat these three colours.
**INVESTIGATE the Math**

Georgia, a fabric artist, has been patterning with equilateral triangles. Consider Georgia’s conjecture about the following pattern.

I think Figure 10 in this pattern will have 100 triangles, and all these triangles will be congruent to the triangle in Figure 1.

**How did Georgia arrive at this conjecture?**

A. Organize the information about the pattern in a table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Triangles</td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. With a partner, discuss what you notice about the data in the table.

C. Extend the pattern for two more figures.

D. What numeric pattern do you see in the table?

**Answers**

A. The pattern in the table shows that the number of triangles equals the square of the figure number.

C.

D. Figure 11 has $11^2$ or 121 triangles. Figure 12 has $12^2$ or 144 triangles.

The numeric pattern in the table shows that each figure will have a perfect square of congruent triangles. The number of congruent triangles in each figure is the square of the figure number.
Reflecting

E. Is Georgia’s conjecture reasonable? Explain.

F. How did Georgia use inductive reasoning to develop her conjecture?

G. Is there a different conjecture you could make based upon the pattern you see? Explain.

Answers

E. Georgia’s conjecture is reasonable because, when the table is extended to the 10th figure, the pattern of values is the same as Georgia’s prediction.

F. Georgia used inductive reasoning by gathering evidence about more cases. This evidence established a pattern. Based on this pattern, Georgia made a prediction about what the values would be for a figure not shown in the evidence.

G. A different conjecture could be made because a different pattern could have been seen. If the focus had been only on the congruent triangles with their vertices at the bottom and their horizontal sides at the top, then the following conjecture could have been made: The 5th figure will have 10 congruent triangles.

[Diagrams and table showing the number of triangles and the corresponding figures]
**Example 2**  
Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

**Jay’s Solution**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>((+3)(+7) = (+21))</td>
<td>Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.</td>
</tr>
<tr>
<td>((-5)(-3) = (+15))</td>
<td>Next, I tried two negative odd integers. The product was again positive and odd.</td>
</tr>
<tr>
<td>((+3)(-3) = (-9))</td>
<td>Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.</td>
</tr>
</tbody>
</table>

My conjecture is that the product of two odd integers is an odd integer.

I noticed that each pair of integers I tried resulted in an odd product.

\((-211)(-17) = (+3587)\)  
I tried other integers to test my conjecture. The product was again odd.
**Example 2**  
Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

**Your Turn**

Do you find Jay’s conjecture convincing? Why or why not?

**Answer**

Here are two possible answers:

- Yes. Jay’s conjecture is convincing because all the different combinations with positive and negative odd integers were used as samples. These three samples showed a pattern in their products, which Jay then tested with different integers. Jay’s conjecture was supported by this last sample.

- No. Jay looked at only three cases before he made his conjecture, then tested it with only one more example. This is not a lot of evidence to base a conjecture on.
**EXAMPLE 3**  
**Using inductive reasoning to develop a conjecture about perfect squares**

Make a conjecture about the difference between consecutive perfect squares.

**Steffan’s Solution: Comparing the squares geometrically**

I represented the difference using unit tiles for each perfect square. First, I made a $3 \times 3$ square in orange and placed a yellow $2 \times 2$ square on top. When I subtracted the $2 \times 2$ square, I had 5 orange unit tiles left.

Next, I made $3 \times 3$ and $4 \times 4$ squares. When I subtracted the $3 \times 3$ square, I was left with 7 orange unit tiles. I decided to try greater squares.

I saw the same pattern in all my examples: an even number of orange unit tiles bordering the yellow square, with one orange unit tile in the top right corner. So, there would always be an odd number of orange unit tiles left, since an even number plus one is always an odd number.

My conjecture is that the difference between consecutive squares is always an odd number.

I tested my conjecture with the perfect squares $7 \times 7$ and $8 \times 8$. The difference was an odd number.

The example supports my conjecture.
EXAMPLE 3  Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Francesca’s Solution: Describing the difference numerically

\[ 2^2 - 1^2 = 4 - 1 \]
\[ 2^2 - 1^2 = 3 \]

I started with the smallest possible perfect square and the next greater perfect square: \(1^2\) and \(2^2\). The difference was 3.

\[ 4^2 - 3^2 = 7 \]
\[ 9^2 - 8^2 = 17 \]

Then I used the perfect squares \(3^2\) and \(4^2\). The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

\[ 12^2 - 11^2 = 23 \]

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

The example supports my conjecture.

To test my conjecture, I tried the perfect squares \(11^2\) and \(12^2\). The difference was a prime number.
EXAMPLE 3  Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Your Turn

How is it possible to have two different conjectures about the same situation? Explain.

Answer

It is possible to have two different conjectures about the same situation because different samples were used to develop the conjecture. Francesca used different values for the sizes of consecutive squares. When she examined her evidence, the common feature from her examples was different from the common feature that Steffan found from the evidence he had developed.
## In Summary

### Key Idea
- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

### Need to Know
- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.
5. Marie studied the sum of the angles in quadrilaterals and made a conjecture. What conjecture could she have made?

The sum of the angles in quadrilaterals is 360°

6. Use the evidence given in the chart below to make a conjecture. Provide more evidence to support your conjecture.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>quadrilateral</th>
<th>pentagon</th>
<th>hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fewest Number of Triangles</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

The fewest # of Δs in a polygon is the # of sides subtracted by 2

8. Dan noticed a pattern in the digits of the multiples of 3. He created the following table to show the pattern.

<table>
<thead>
<tr>
<th>Multiples of 3</th>
<th>12</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of the Digits</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

a) Make a conjecture based on the pattern in the table.

The sum of the digits of multiples of 3 are always 3, 6, and 9

11. Paula claims that whenever you square an odd integer, the result is an odd number. Is her conjecture reasonable? Justify your decision.

\[3^2 = 9\]
\[7^2 = 49\]
\[11^2 = 121\]
\[(-3)^2 = 9\]
\[(-7)^2 = 49\]
Math in Action

Oops! What Happened?

- Identify the pieces of given evidence that are conjectures.
- Make a conjecture about what caused the accident.
- What evidence supports your conjecture?
- If you could ask three questions of the drivers or the witness, what would they be?
- Can the cause of an accident such as this be proved? Why or why not?